

Formula Sheet

PHYSICS

CLASS 11

UNO
test prep

Medical
IIT-JEE
Foundations



Physical World

ONLY THEORY



Units and Measurement

★ Determination of Radius of atom using avogadro Hypothesis

$$r = \left(\frac{VM}{2\pi Nm} \right)^{1/3}$$

V = Volume
M = Molecular weight
N = Avogadro's No.

★ Diameter of Moon

$$D = 50$$

θ = Angle of Deviation

$$\star \text{Angle} = \frac{\text{Curvle Radius}}{\text{Radius}}$$

★ Absolute Error

$$\Delta a_n = a_{\text{mean}} - a_n$$

$$a_{\text{mean}} = \sum_{i=1}^n a_i / n$$

★ Mean Absolute Error

$$\Delta a_{\text{mean}} \quad \bar{\Delta a} = \frac{1}{n} \sum_{i=1}^n |\Delta a_i|$$

★ Relative or Fractional Error

$$= \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$$

★ Percentage Error

$$\delta a = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$$

★ Error in addition and Subtraction

$$\Delta z_{\text{max}} = \Delta A + \Delta B$$

★ Error in Multiplication and Division

$$\left| \frac{\Delta z}{z} \right|_{\text{max}} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

★ Error in a power of measured quantity

$$\text{if } z = A^m$$

$$\text{if } z = \frac{A^m B^n}{C^p}$$

$$\left| \frac{\Delta z}{z} \right|_{\text{max}} = m \frac{\Delta A}{A}$$

$$\left| \frac{\Delta z}{z} \right|_{\text{max}} = m \frac{\Delta A}{A} + n \frac{\Delta B}{B} + p \frac{\Delta C}{C}$$

★ Experimental %. Error

$$\frac{\text{Real Value} - \text{Experimental Value}}{\text{Real Value}} \times 100\%$$



Motion in a straight line

★ Speed $v = \frac{s}{t}$

$$v = \frac{s}{t}$$

★ Average speed $\bar{v} = \frac{\Delta s}{\Delta t}$

$$\bar{v} = \frac{\Delta s}{\Delta t}$$

★ Average velocity

$$\vec{v}_{av} = \frac{\vec{s}}{\Delta t}$$

★ Instantaneous speed

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

s = displacement, t = time

v = initial velocity

u = final velocity

★ Instantaneous velocity

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{s}}{\Delta t} = \frac{d\vec{s}}{dt}$$

★ Acceleration

$$a = \frac{\Delta v}{\Delta t}$$

$$\vec{v}_{av} = \frac{\vec{u} + \vec{v}}{2}$$

OR

$$\vec{v}_{av} = \frac{\vec{v}_1 t_1 + \vec{v}_2 t_2}{t_1 + t_2}$$

★ Instantaneous Acceleration

$$\vec{a} = \frac{d(\vec{v})}{dt}$$

$$a = \frac{d^2 s}{dt^2}$$

★ Equations of Motion or Equations of Kinematics

$$1. v = v_0 + at$$

(Velocity-time Relation)

$$2. s = v_0 t + \frac{1}{2} a t^2$$

(Position-Time Relation)

$$3. v^2 = v_0^2 + 2as$$

(Position-velocity Relation)

★ Motion under gravity

(A) When down to up

$$s = h, a = g$$

$$1. v = v_0 - gt$$

$$2. h = v_0 t - \frac{1}{2} g t^2$$

$$3. v^2 = v_0^2 - 2gh$$

★ (B) When up to down

$$s = h, a = g$$

$$1. v = v_0 + gt$$

$$2. h = v_0 t + \frac{1}{2} g t^2$$

$$3. v^2 = v_0^2 + 2gh$$

★ Stopping distance of vehicles

$$d_s = -\frac{v_0^2}{2a}$$

★ Distance traveled in n^{th} sec

$$\Delta s = v_0 + \frac{1}{2} a (2n-1)$$

★ Relative velocity A to B

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

$$\vec{V}_{AB} = \vec{V}_A + \vec{V}_B$$

$$\vec{V}_{AB} = -\vec{V}_{BA}$$

(Object move in the opposite direction)



Motion in a Plane

★ Equal vectors

$$\vec{A} = \vec{B} = \vec{C}$$

★ Opposite vectors

$$\vec{A} = -\vec{D}$$

★ Unit vector

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}$$

★ Orthogonal unit vector

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

★ Magnitude

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

★ Vector addition

$$\vec{R} = \vec{A} + \vec{B}$$

★ Vector subtraction

$$\vec{R} = \vec{A} - \vec{B}$$

★ Analytical Method

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

★ Resolution of Vectors

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1} \frac{A_y}{A_x}$$

★ Properties of Vectors

$$1. |\lambda \vec{A}| = |\lambda| |\vec{A}|$$

If $\lambda > 0$ (λ = real no.)

$$8. \vec{0}\vec{A} = \vec{0}$$

$$2. \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

$$3. \vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$9. \lambda \vec{0} = \vec{0}$$

$$4. (\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

$$5. \vec{A} - \vec{A} = \vec{0} \quad |\vec{0}| = 0 \text{ (zero vector)}$$

$$6. \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

$$7. \vec{A} + \vec{0} = \vec{A}$$

★ Scalar or dot products of two vectors

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

★ Properties of Scalar Product

$$1. (a) \theta < 90^\circ$$

$$\vec{A} \cdot \vec{B} \text{ (+ve)}$$

$$(b) \theta = 90^\circ$$

$$\vec{A} \cdot \vec{B} \text{ (zero)}$$

$$(c) \theta > 90^\circ$$

$$\vec{A} \cdot \vec{B} \text{ (-ve)}$$

$$2. \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \quad \text{(commutative)}$$

$$3. \vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C} \quad \text{(Distributive)}$$

$$4. \vec{A} \cdot \vec{B} = 0 \quad \theta = 90^\circ \quad \text{(two perpendicular vectors)}$$

5. $\vec{A} \cdot \vec{B} = AB$ $\theta = 0^\circ$ (two parallel vectors)
 $\vec{A} \cdot \vec{B} = -AB$ $\theta = 180^\circ$

6. $\vec{A} \cdot \vec{A} = A^2$ (Product of a vector with itself is equal to square)

7. $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$ $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ (unit orthogonal vector relations)

8. $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$ (two vectors is equal to the sum of the product)

★ Vector or cross product of two vectors $\vec{A} \times \vec{B} = AB \sin\theta \hat{n}$

★ Properties of Vector product

1. $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ (not commutative)

2. $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$ (Distributive)

3. $(m\vec{A}) \times \vec{B} = \vec{A} \times (m\vec{B}) = mAB \sin\theta \hat{n}$ (two vector is multiplied by scalar)

4. $|\vec{A} \times \vec{B}| = AB$ $\theta = 90^\circ$ (two perpendicular vectors)

5. $\vec{A} \times \vec{B} = 0$ $\theta = 0^\circ$ (two parallel vectors)

6. $\vec{A} \times \vec{A} = 0$ (Product of a vector by itself)

7. $\hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = \hat{k}$ $\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i}$ $\hat{k} \times \hat{i} = -\hat{i} \times \hat{k} = \hat{j}$

$\hat{i} \times \hat{k} = \hat{j}$ $\hat{j} \times \hat{i} = \hat{k}$

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

where $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

★ Motion in a plane

1. Position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

2. Displacement vector $\Delta \vec{r} = \vec{r}' - \vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

3. Velocity $\vec{v} = \frac{\Delta \vec{r}}{\Delta t}$ $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$

4. Acceleration $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$ $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$ $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

★ Motion in plane with constant velocity

$\vec{r} = \vec{r}_0 + \vec{v}t$

★ Motion in plane with constant Acceleration

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

In direction of X-axis

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

★ Projectile Motion

horizontal component

$$u_x = v_0 \cos \theta_0$$

Vertical Component

$$u_y = v_0 \sin \theta_0$$

↳ Path of Projectile

$$y = (\tan \theta_0) x - \frac{g}{2v_0^2 \cos^2 \theta_0} x^2$$

↳ Flight time of Projectile

$$t = \frac{v_0 \sin \theta_0}{g}$$

$$T = 2t = \frac{2v_0 \sin \theta_0}{g}$$

↳ Height of Projectile

$$h = \frac{v_0^2 \sin^2 \theta_0}{2g}$$

$$h_{\max} = \frac{v_0^2}{2g}$$

↳ Range of Projectile

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

If $\theta = 45^\circ$

$$R_{\max} = \frac{v_0^2}{g}$$

★ Uniform circular motion

$$a_c = \frac{v^2}{R}$$

$$a_c = \omega^2 R$$

a_c = Centripetal force

R = Radius of circle

v = frequency

$$a_c = 4\pi^2 f^2 R$$

★ Angular velocity

$$\omega = \frac{d\theta}{dt}$$

$$\omega = 2\pi f$$

$$v = 2\pi R f$$

★ Relation between v and ω

$$v = \omega R$$

v = linear velocity

ω = angular velocity

$\therefore (v = \omega R)$

★ Centripetal force

$$F = \frac{mv^2}{R}$$

$$F = m\omega^2 R$$

★ Motion of a conical pendulum

$$\text{time period (T)} = 2\pi \sqrt{\frac{l \cos \theta}{g}}$$

★ Motion of particle tied to a string in a vertical circle

$$v = \sqrt{(3 + 2 \cos \theta) gl}$$

$$T = 3mg(1 + \cos \theta)$$

1. If $\theta = 90^\circ$

$$v = \sqrt{5gl}$$

$$T = 6mg$$

(At bottom of circle)

2. If $\theta = 90^\circ$

$$v = \sqrt{3gl}$$

$$T = 3mg$$

3. If $\theta = 180^\circ$

$$v = \sqrt{gl}$$

$$T = 0$$

(At top of circle)



Laws of Motion

★ Momentum

$$p = m \times v$$

★ Impulse

$$J = mv$$

★ Force

$$F = ma = \frac{dp}{dt}$$

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

★ Law of Conservation of Momentum

$$\vec{p} = \text{constant} \quad [\text{if } F_m = 0]$$

★ Equilibrium of a particle

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

★ Limiting friction

$$f_s = \mu_s R \quad \mu_s = \text{coefficient of static friction}$$

★ Kinetic friction

$$f_k = \mu_k R \quad \mu_k = \text{coefficient of kinetic friction}$$

★ Angle of friction

$$\tan \theta_s = \mu_s$$

★ Friction on an inclined Plane

$$\mu_s = \tan \theta_s$$

$$\mu_k = \tan \theta_k$$

$$\mu_s = \frac{f_s}{R} = \frac{f_s}{mg}$$

$$f = \frac{mv^2}{R}$$

★ Motion of a car on a level road

$$v_{max} = \sqrt{\mu_s R g}$$

★ Motion of a car on a banked road

$$v_{max} = \left[Rg \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta} \right]^{1/2}$$



Work, Energy And Power

★ Work Energy Theorem

$$K_f - K_i = W \quad \text{OR} \quad \Delta K = W$$

★ Work

$$W = \vec{F} \cdot \vec{s}$$

★ Kinetic Energy

$$K = \frac{1}{2} mv^2$$

★ Work done by a variable force

$$W = \int_{x_i}^{x_f} F dx$$

★ Power

$$P = \frac{dW}{dt} \quad \text{OR} \quad P = Fv$$

★ Gravitational Potential Energy

$$U = mgh$$

★ Potential Energy of spring

$$U = \frac{1}{2} kx^2$$

★ Conservative Forces as Negative gradient of Potential Energy

$$\frac{dU}{dx} = -F \quad \text{OR} \quad \frac{dU}{dx} = F'$$

$$E = mc^2$$

★ Equivalence of Mass and Energy

★ Conservation of Energy

$$\Delta K + \Delta U = 0 \quad \text{OR} \quad \Delta E = 0$$

★ Elastic collision in 1D

$$u_1 - u_2 = v_2 - v_1 = -(v_1 - v_2)$$

$$v_1 = \left[\frac{m_1 - m_2}{m_1 + m_2} \right] u_1 + \left[\frac{2m_1}{m_1 + m_2} \right] u_2$$

$$v_2 = \left[\frac{2m_1}{m_1 + m_2} \right] u_1 + \left[\frac{m_2 - m_1}{m_1 + m_2} \right] u_2$$

1. If $m_1 = m_2 = m$

★ Elastic collision in 2D

Along X-axis,

$$m_1 u_1 = m_1 v_1 \cos\theta + m_2 v_2 \cos\theta$$

Then $v_1 = u_1$ and $v_2 = u_2$

Along Y-axis,

$$0 = m_1 v_1 \sin\theta + m_2 v_2 \sin\theta$$

$$m_1 u_1^2 = m_1 v_1^2 + m_2 v_2^2$$

2. If m_2 is in rest i.e. $u_2 = 0$

★ Perfectly inelastic collision

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2} \right) u_1$$

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

$$\frac{K_2}{K_1} = \frac{m_1}{m_1 + m_2}$$



System of Particle And Rotational Motion

★ Centre of Mass of a two - particle system

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

★ Centre of Mass of a system of n - particles

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n}{m_1 + m_2 + \dots + m_n}$$

★ Centre of Mass of a Rigid body

$$\vec{r}_{cm} = \frac{L}{M} \int \vec{r} dm$$

★ Centre of Mass of a uniform rod

$$x_{cm} = \frac{L}{2}$$

★ Motion of the centre of mass

$$\vec{v}_{cm} = \frac{1}{M} (m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n)$$

and $M \vec{a}_{cm} = \vec{F}_{ext}$

★ Momentum conservation

$$\vec{P} = M \vec{v}_{cm}$$

★ Centre - of - Mass motion

$$\frac{d\vec{P}}{dt} = \vec{F}_{ext}$$

★ Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

★ Acceleration of centre of Mass

$$\vec{a}_{cm} = \frac{\vec{F}}{M}$$

★ Angular Acceleration

$$\alpha = \frac{d\omega}{dt}$$

$$a = r\alpha$$

$$v = rw$$

★ Equations of Rotational Motion

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

★ Moment of Inertia

$$I = \sum m r_i^2$$

★ Radius of Gyration

$$k = \sqrt{\frac{I}{M}}$$

★ Relation between τ and α

$$\vec{\tau} = I \times \vec{\alpha}$$

★ Theorem of Parallel axis

$$I = I_{cm} + Ma^2$$

★ Theorem of Perpendicular axis

$$I_z = I_y + I_x$$

★ Relation between L and I

$$\vec{L} = \vec{I} \times \vec{\omega}$$

★ Rate of Change of Angular Momentum

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{ext}$$

★ Law of Conservation of Angular momentum

$$\vec{L} = \text{constant} \quad (\text{if } \vec{\tau}_{ext} = 0)$$

★ Equilibrium of rigid body

$$1) \sum_{i=1}^n \vec{F}_i = 0$$

$$2) \sum_{i=1}^n \vec{T}_i = 0$$

Some of all forces & Torque must be zero.

★ Kinetic Energy of Rotation

$$K = \frac{1}{2} I w^2$$

★ Condition of Rolling of a body without sliding over an inclined Plane

$$f_s = \frac{I a}{R^2}$$

$$a = \frac{g \sin \theta}{\left(1 + \frac{K^2}{R^2}\right)}$$

(i) If the rolling body is a solid cylinder

$$\mu_s = \frac{1}{3} \tan \theta$$

(ii) If the rolling body is a solid Sphere

$$\mu_s = \frac{2}{7} \tan \theta$$

★ Total Energy of a body Rolling without slipping

$$K_{total} = K_{rot} + K_{trans}$$

$$and \quad K_{total} = \frac{1}{2} I w^2 + \frac{1}{2} M v^2$$

★ Rolling Motion

$$V_{cm} = R w$$

1. Rolls without slip $V_{cm} = R w$

2. Rolls with slipping in forward direction $V_{cm} > R w$

3. Rolls with slipping in Backward direction $V_{cm} < R w$



Gravitation

★ Kepler's law

First law (law of orbits) Planets move in elliptical orbits around sun

Second law (Law of Areal velocity) $\frac{dA}{dt} = \frac{I}{2m}$

Third law (Law of Periods) $T^2 \propto a^3$

★ Universal law of Gravitation

$$F = G \frac{m_1 m_2}{r^2}$$

Vector form,

$$\vec{F} = -G \frac{m_1 m_2 \hat{r}}{|R|^3}$$

$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$ (Universal Gravitational Constant)

★ Intensity of Gravitational field

$$\vec{I} = \frac{\vec{F}}{m}$$

★ Relation between 'g' and 'G'

$$g = \frac{G M_e}{R_e^2}$$

g = Acceleration due to earth's gravity

M_e = Mass of Earth

R_e = Radius of Earth

ρ = Density of Earth

★ Computation of Mass and Density of Earth

$$M_e = \frac{g R_e^2}{G}$$

$M_e = 6.0 \times 10^{24}$

$$\rho = \frac{3g}{4\pi R_e G}$$

$\rho = 5.5 \times 10^3 \text{ kg/m}^3$

★ Variation in Acceleration due to gravity above the surface of the earth

$$g' = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2}$$

★ Variation in Acceleration due to gravity below the surface of the earth

$$g' = g \left(1 - \frac{h}{R_e}\right)$$

★ Gravitational Potential

$$V = -\frac{W}{m}$$

OR $V = -\frac{G M}{r}$

★ Gravitational Potential Energy

$$U = -\frac{G M m}{r}$$

★ Speed of Satellite

$$v_o = \sqrt{g R_e}$$

v_o = orbital velocity

★ Period of Revolution of Satellite

$$T = \sqrt{\frac{3\pi}{GP}}$$

★ Total Energy of Satellite

$$E = -\frac{1}{2} \frac{G M_e m}{R_e}$$

★ Binding Energy of a Satellite $= + \frac{1}{2} \frac{G M_e m}{R_e} = + \frac{1}{2} m g R_e$

★ Geostationary Satellite $R_e + h = \left(\frac{T^2 g R_e}{4\pi^2} \right)^{1/3}$ for $T = 24$ hours
 $h = 35745$ km

★ Escape Energy $= + \frac{G M_e m}{R_e}$ Escape velocity $v_e = \frac{2 G M_e}{R_e}$

★ Relation between v_0 and v_e $v_e = \sqrt{2} v_0$ OR $v_e = \sqrt{2 g R_e}$

UNOTESTPREP



Mechanical Properties of Solids

★ Stress = $\frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$

★ Strain

Longitudinal strain = $\frac{\Delta L}{L}$

★ Hooke's Law

$$E = \frac{\text{STRESS}}{\text{STRAIN}}$$

E = Modulus of Elasticity

Shearing strain = $\frac{\Delta x}{L} = \tan\theta$

Volume strain = $\frac{\Delta V}{V}$

★ Young's Modulus of Elasticity

$$\gamma = \frac{\sigma}{E} = \frac{Mg L}{\pi r^2 \Delta L}$$

σ = longitudinal stress
E = longitudinal strain

$\gamma_{\text{Steel}} > \gamma_{\text{Rubber}}$

★ Bulk Modulus of Elasticity

$$B = \frac{\text{Normal Stress}}{\text{Volume Strain}} = -\frac{PV}{\Delta V}$$

★ Compressibility

$$\beta = \frac{1}{B} = -\frac{\Delta V}{PV}$$

★ Modulus of Rigidity

$$\eta = \frac{F}{AB}$$

★ Poisson's Ratio

$$\sigma = \frac{\Delta h L}{\Delta L h}$$

★ Force produced in cooling a wire stretched between two rigid supports

$$F = YA \alpha \Delta t$$

★ Work done in stretching a wire and Elastic potential Energy

$$W = U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$W = \frac{1}{2} \frac{YA}{L} x^2$$

★ Energy density

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

and $U = \frac{1}{2} \times \text{Young's Modulus} \times (\text{Strain})^2$



Mechanical Properties of fluids

★ Pressure

$$P = \frac{F}{A}$$

$$P = \frac{dF}{dA}$$

★ Volume density

$$\rho = \frac{m}{V}$$

★ Effect of gravity on fluid pressure

$$P = P_1 + \rho gh$$

η = viscosity
D = Diameter of pipe
 R_e = Reynold No.

★ Critical velocity of a liquid

$$v_c = \frac{R_e \eta}{PD}$$

★ Reynold number

$$R_e = \frac{\rho v D}{\eta}$$

★ Coefficient of viscosity

$$F = \pm \eta A \frac{\Delta V_x}{\Delta z}$$

$$\eta = \frac{F l}{V A}$$

★ Terminal velocity

$$v = \frac{2 \eta^2 (P - \sigma) g}{9 \eta}$$

★ Kinetic Energy

$$K = \frac{1}{2} P V^2$$

★ Potential Energy

$$U = \rho g h$$

★ Principle of Continuity

$$A \times v = \text{constant}$$

★ Stoke's Law

$$F = 6 \pi \eta r v$$

★ Bernoulli's Theorem

$$P + \frac{1}{2} P V^2 + \rho g h = \text{constant}$$

OR

$$P_1 + \frac{1}{2} P V_1^2 + \rho g h_1 = P_2 + \frac{1}{2} P V_2^2 + \rho g h_2$$

$$\frac{P}{\rho g} + \frac{V^2}{2g} + h = \text{constant}$$

$\frac{P}{\rho g}$ = Pressure head, $\frac{V^2}{2g}$ = Velocity head

$$P + \frac{1}{2} P V^2 = \text{constant}$$

$$g_f(h_1 = h_2)$$

h = gravitational head

★ Venturiometer

$$V_1 = \sqrt{\frac{2 P_m \gamma h}{P} \left[\left(\frac{A}{a} \right)^2 - 1 \right]^{-1/2}}$$

★ Velocity Gradient = $\frac{\Delta V}{\Delta z}$

★ Velocity of Efflux of a Liquid

$$v = \sqrt{2gh}$$

(Torricelli's theorem)

$$t = \sqrt{\frac{2(H-h)}{g}}$$

$$x = 2\sqrt{h(H-h)}$$

$$x_{\max} = H$$

H = height of fluid level
h = height of nozzle from top

★ Work done in increasing the surface area of free surface of the liquid

$$F = T \times 2l$$

$$W = T \times 2l \times \Delta x$$

$$W = T \times \Delta A$$

T = Surface Tension

ΔA = Increment in surface area

★ Excess of Pressure

(i) Excess of Pressure Inside a liquid drop

$$P = \frac{2T}{R}$$

(ii) Excess of Pressure inside in an air bubble in a liquid

$$P = \frac{2T}{R}$$

(iii) Excess of Pressure inside a bubble of a Soap Solution

$$P = \frac{4T}{R}$$

★ Capillary

$$hpg = \frac{2T}{R}$$

$$h = \frac{2T \cos \theta}{\rho g}$$

r = radius of capillary
R = Radius of meniscus

Thermal Properties of Matter

★ Scale of Temperature

$$T_F = 32 + \frac{9}{5} T_C$$

★ Ideal gas Equation For 1 mole

$$PV = RT$$

For n mole

$$PV = nRT$$

★ Absolute Temperature

$$T = T_C + 273.15$$

★ Thermal Expansions of Solids

Coefficient of linear Expansion

$$\alpha = \frac{\Delta L}{L \times \Delta t}$$

Coefficient of Superficial Expansion

$$\beta = \frac{\Delta A}{A \times \Delta t}$$

Coefficient of Volume Expansion

$$\gamma = \frac{\Delta V}{V \times \Delta t}$$

★ Relation between α and β

$$\beta = 2\alpha$$

★ Relation between γ and α

$$\gamma = 3\alpha$$

★ Relation between β and γ

$$3\beta = 2\gamma$$

★ Relation between α , β and γ

$$\alpha : \beta : \gamma = 1 : 2 : 3$$

★ Variation of density with temperature of liquids

$$d_t = d_0(1 - \gamma_h t)$$

★ Heat capacity

$$S = \frac{\Delta Q}{\Delta t}$$

★ Specific heat capacity

$$S = \frac{Q}{m} = \frac{1}{m} \frac{\Delta Q}{\Delta t}$$

★ Heat Capacity per mole

$$C = \frac{S}{n} = \frac{1}{n} \frac{\Delta Q}{\Delta t}$$

★ Calorimeter

$$W = mc$$

★ Principal of Calorimeter

★ Latent Heat

$$L = \frac{m}{Q}$$

$$\text{Heat lost} = \text{Heat taken}$$

OR

$$m_1 c_1 (t_1 - t) = m_2 c_2 (t - t_2)$$

★ Coefficient of Thermal conductivity

$$Q = KA \frac{(0_1 - 0_2)t}{l}$$

Q = Heat flowed

★ Rate of Heat flowed

OR

Heat current

$$H = \frac{Q}{t} = KA \frac{0_1 - 0_2}{l}$$

★ Thermal Radiation

$$I \propto \frac{1}{A^2}$$

$$\Lambda_{\text{radiation}} > \Lambda_{\text{light}}$$

★ Thermal Resistance

$$R = \frac{\theta_1 - \theta_2}{H} = \frac{l}{KA}$$

★ Emissivity

$$\epsilon = \frac{Q}{AXt}$$

★ Kirchoff's law

$$\frac{e_A}{a_A} = E_A$$

e_A = Spectral Emissive power

a_A = Spectral absorption power

E_A = Emissive power of black body

σ = Stefan's constant

★ Stefan's law

$$E = \sigma T^4$$

★ Newton's law of cooling

$$-\frac{d\theta}{dt} = K(T_2 - T_1)$$

$$\text{and } \log_e(T_2 - T_1) = -KT + c$$

★ Wein's law

$$\lambda_m \times T = b$$

λ_m = wave length

T = Temperature

Note :-

1. $T_{\text{low}} \leq T_{\text{mixture}} \leq T_{\text{High}}$

2. $T_{\text{mixture}} = \frac{m_1 s_1 T_1 + m_2 s_2 T_2}{m_1 s_1 + m_2 s_2}$



Thermodynamics

★ Equivalence of Work and heat

$$W = JH \quad J = \text{mechanical equivalent of heat}$$

★ Work

$$W = P(V_2 - V_1) \quad \text{OR} \quad \Delta W = P\Delta V$$

1 Kilocalori = 4.18×10^3 Joule

★ First law of thermodynamics

$$\Delta U = \Delta Q - \Delta W \quad \text{OR} \quad \Delta U = \Delta Q - P\Delta V$$

★ Mayer's formula

$$C_p - C_v = R$$

★ Isothermal process (T constant)

$$C_v = \left(\frac{\Delta Q}{\Delta T} \right)_v = \left(\frac{\Delta U}{\Delta T} \right)_v$$

$$W = uRT \ln \frac{V_2}{V_1}$$

$$C_p = \left(\frac{\Delta Q}{\Delta T} \right)_p = \left(\frac{\Delta U}{\Delta T} \right)_p + P \left(\frac{\Delta V}{\Delta T} \right)_p$$

★ Isochoric process $V = \text{constant}$

★ Adiabatic Process

$$PV^\gamma = \text{constant}$$

★ Isobaric process $P = \text{constant}$

$$\gamma = \frac{C_p}{C_v}$$

$$TV^{\gamma-1} = \text{constant}$$

$$W = P(V_2 - V_1) = uR(T_2 - T_1)$$

$$W = uR \frac{(T_1 - T_2)}{\gamma - 1}$$

$$\frac{T^\gamma}{P^{\gamma-1}} = \text{constant}$$

★ Cyclic Process

$$\Delta U = 0$$

★ Total Internal Energy of an ideal gas

$$U = \frac{1}{2} f R T$$

$$C_V = \frac{1}{2} f R$$

$$C_P = \left(\frac{f}{\vartheta} + 1 \right) R$$

$$\gamma = 1 + \frac{2}{f}$$

★ Second law of thermodynamics

$$W = Q_1 - Q_2$$

$$\eta = \frac{W}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$\text{OR} \quad \eta = 1 - \frac{T_2}{T_1}$$

$$\therefore \frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

★ Heat Engine

1. Isothermal Expansion

$$W_1 = RT_1 \log_e \frac{V_2}{V_1} \quad (T \text{ constant})$$

2. Adiabatic Expansion

$$W_2 = \frac{R}{1-\gamma} (T_2 - T_1) \quad (PV^\gamma = \text{constant})$$

3. Isothermal compression

$$W_3 = -RT_2 \log_e \frac{V_3}{V_4}$$

4. Adiabatic compression

$$W_4 = \frac{R}{1-\gamma} (T_1 - T_2)$$



Kinetic Theory

★ Boyle's law At constant T $V \propto \frac{1}{P}$ OR $PV = \text{Constant}$ $P_1 V_1 = P_2 V_2$

★ Charles' law At constant P $V \propto T$ OR $\frac{V}{T} = \text{Constant}$ $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

★ Ideal Gas Equation $PV = uRT$ For u mole $PV = RT$ For 1 mole
 $R = 8.31 \text{ J/mol}\cdot\text{K}$ (Universal Gas Constant)

★ Dalton's law of Partial Pressure $P = \frac{u_1 RT}{V} + \frac{u_2 RT}{V} + \dots = P_1 + P_2 + \dots$

★ Formula for the pressure of an ideal gas $P = \frac{1}{3} mn \bar{v}^2$ OR $P = \frac{1}{3} \rho \bar{v}^2$

★ Root mean square velocity $\bar{v}^2 = \frac{3RT}{M}$ $v_{rms} = \sqrt{\frac{3RT}{M}}$ $v_{rms} \propto \sqrt{T}$

★ Kinetic Interpretation of Temperature $\frac{E}{N} = \frac{3}{2} K_B T$ If $n=1$ $E = \frac{3}{2} K_B T$ (K_B is per 1 atom)

★ Real gas Equation $\left(P + \frac{a}{v^2}\right) (V - b) = RT$

★ Graham's law of diffusion $\frac{v_{1rms}}{v_{2rms}} = \frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$ R = Rate of diffusion

★ Law of Equipartition of Energy

$$E_t = \frac{1}{2} mv_x^2 + \frac{1}{2} mv_y^2 + \frac{1}{2} mv_z^2$$

$$E_{xyz} = \frac{3}{2} K_B T \quad K_B = \text{Boltzmann's constant}$$

★ Specific Heat Capacity

1) Monoatomic Gas $C_p = \frac{5}{2} R$ $C_v = \frac{3}{2} R$ $\gamma = \frac{5}{3}$

2) Diatomic Gas $C_p = \frac{7}{2} R$ $C_v = \frac{5}{2} R$ $\gamma = \frac{7}{5}$

3) Polyatomic Gas $C_p = (4+f)R$ $C_v = (3+f)R$ $\gamma = \frac{4+f}{3+f}$

4) Specific Heat capacity of Solids $C = 3R$

5) Specific Heat capacity of water $C = 9R$

★ Mean free path $\lambda = \frac{1}{\sqrt{2} \pi d^2 n}$

for N atoms $\lambda = \frac{K_B T}{\sqrt{2} \pi m} \cdot \frac{N}{n} \cdot \frac{1}{d^3}$

Oscillations

★ Relation between v and T
v : frequency

$$v = \frac{1}{T}$$

★ Time period

$$T = \frac{2\pi}{\omega}$$

★ Displacement Equation of Simple Harmonic Motion (S.H.M.)

$$x(t) = A \cos(\omega t + \phi)$$

$$y(t) = a \sin(\omega t + \phi)$$

★ Velocity in S.H.M.

$$v(t) = -\omega A \sin(\omega t + \phi)$$

$$v(t) = \frac{d}{dt} x(t)$$

★ Acceleration in S.H.M.

$$\alpha = -\omega^2 x(t)$$

★ Time - Displacement curve of S.H.M.

$$y = a \sin\left[\frac{2\pi t}{T}\right]$$

★ Force law in Simple Harmonic motion (Spring)

$$F = -kx$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = 2\pi \sqrt{\frac{m}{2k}}$$

ω : Angular frequency
 T : Time period

★ Potential Energy in S.H.M.

$$U = \frac{1}{2} m \omega^2 y^2$$

★ Kinetic Energy in S.H.M.

$$K = \frac{1}{2} m \omega^2 (a^2 - y^2)$$

★ Total Energy in S.H.M.

$$E = \frac{1}{2} m \omega^2 a^2 = 2\pi^2 m n^2 a^2$$

★ Time period of S.H.M.

$$T = 2\pi \sqrt{\frac{m}{k}}$$

★ Motion of body suspended by two springs

$$T = 2\pi \sqrt{\frac{m}{K_1 + K_2}}$$

$$T = 2\pi \sqrt{m \left(\frac{1}{K_1} + \frac{1}{K_2} \right)}$$

★ Time period of Simple Pendulum

$$T = 2\pi \sqrt{\frac{l}{g}}$$

★ Second's Pendulum

$$l = \frac{g}{\pi^2} = 1 \text{ m}$$

R_E : Radius of Earth ($6.4 \times 10^6 \text{ m}$)

★ Time period of a simple pendulum of infinite length

$$T = 2\pi \sqrt{\frac{R_E}{g}}$$

★ Electromagnetic Resonance frequency

$$f = \frac{1}{2\pi\sqrt{LC}}$$

★ Damped Simple Harmonic motion $x(t) = A e^{-bt/2m} \cos(\omega' t + \phi)$

$$\omega' = \sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}$$

$$E(t) = \frac{1}{2} k A^2 e^{-bt/m}$$

ω' = angular frequency

★ Forced Oscillations

$$F(t) = F_0 \cos \omega_d t$$

$$x(t) = A \cos(\omega_d t + \phi)$$

$$A = \frac{F_0}{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2 / 4}$$

ω = natural frequency

ω_d = driven frequency

A = Amplitude

(a) Small Damping, Driving frequency far from natural frequency

$$A = \frac{F_0}{m(\omega^2 - \omega_d^2)}$$

(b) Driving frequency close to Natural frequency

$$A = \frac{F_0}{\omega_d b}$$

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Waves

★ Wave Speed

$$v = \frac{\lambda}{T}$$

OR

$$v = n\lambda$$

★ Time period

$$T = \frac{2\pi}{\omega}$$

★ Wave number

$$\bar{v} = \frac{1}{\lambda}$$

★ Angular frequency

$$\omega = 2\pi n$$

★ Propagation constant

$$K = \frac{2\pi}{\lambda}$$

$$v = \frac{\omega}{K}$$

★ Frequency

$$v = \frac{\omega}{2\pi} = \frac{1}{T}$$

★ Wave Equation

$$y(x, t) = A \sin(Kx \pm \omega t + \phi)$$

$$y(x, t) = A \sin(Kx - \omega t) + B \cos(Kx - \omega t)$$

$$a = \sqrt{A^2 + B^2}$$

$$\tan \theta = \frac{B}{A}$$

★ Speed of transverse wave in solid

$$V = \sqrt{\frac{T}{\eta}}$$

η - rigidity coefficient

★ Speed of transverse wave in stretched string

$$V = \sqrt{\frac{T}{\mu}}$$

T - tension of the string
 μ - linear mass density

★ Speed of longitudinal wave in liquid

$$V = \sqrt{\frac{\beta}{P}}$$

β - bulk modulus

★ Speed of longitudinal wave in solid

$$V = \sqrt{\frac{Y}{P}}$$

Y - Young's modulus

★ Speed of longitudinal wave in gases
OR sound in gases

$$V = \sqrt{\frac{P}{\rho}}$$

P - initial pressure

★ Laplace connection

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

ρ - density

★ Effect of temperature on the speed of longitudinal wave

$$V = \frac{\gamma RT}{M}$$

OR

$$\frac{V_1}{V_2} = \sqrt{\frac{M_2}{M_1}}$$

★ Relation between V and v_{rms}

$$V = \left(\sqrt{\frac{\gamma}{3}} \right) v_{rms}$$

★ Principle of Superposition

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

★ Relation between Phase difference and Path difference of two particles in progressive wave

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$$

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta t$$

★ Velocity Amplitude and Acceleration amplitude of a particle in a progressive wave

↳ Velocity Amplitude $U_{max} = \omega a$

↳ Acceleration Amplitude $f_{max} = -\omega^2 a$

★ Equation of Stationary wave

$$y = 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi t}{T}$$

$$y = -2a \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi t}{T}$$

★ Beat frequency

$$v_{beat} = v_1 - v_2$$

★ Modes of vibration of Air column in closed organ pipe

$$\lambda = \frac{4L}{2m-1}$$

1. first mode of vibration $m=1$, $\lambda_1 = 4L$

$$f_1 = \frac{v}{4L}$$

2. Second mode of vibration $m=2$, $\lambda_2 = \frac{4L}{3}$

$$f_2 = \frac{3v}{4L}$$

$$\Rightarrow f_2 = 3f_1$$

3. Third mode of vibration $m=3$, $\lambda_3 = \frac{4L}{5}$

$$f_3 = \frac{5v}{4L}$$

$$\Rightarrow f_3 = 5f_1$$

$$f_1 : f_2 : f_3 \dots = 1 : 3 : 5 \dots$$

★ Modes of vibration of Air column in open organ pipe

$$\lambda = \frac{2L}{m}$$

$$f_m = \frac{v}{\lambda_m} = \frac{mv}{2L}$$

$$f_1 : f_2 : f_3 = 1 : 2 : 3$$

★ End correction

Open Pipe

$$f = \frac{v}{2(L+1.2n)}$$

Closed Pipe

$$f = \frac{v}{4(L+0.6n)}$$

★ Doppler Effect

1. source moving observer stationary

$$v = v_0 \left(1 + \frac{v_s}{v} \right)$$

$$v = v_0 \left(1 - \frac{v_s}{v} \right)$$

2. Observer moving source stationary

$$v = v_0 \left(1 + \frac{v_o}{v} \right)$$

$$v = \text{frequency of wave}$$

$$v_0 = \text{frequency at } L=50$$

$$v = \text{velocity of wave}$$

$$v_s = \text{velocity of source}$$

$$v_o = \text{velocity of observer}$$

3. Both source and observer moving

$$v = v_0 \left(\frac{v+v_o}{v+v_s} \right)$$